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RECOVERY OF ANALOG SIGNAL SHAPES FROM DISCRETE READINGS

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ANNOTATION

A signal model is introduced with the help of a set of possible correlation (spectral) functions. Recovery of the shape of a continuous signal from discrete readings with the help of linear filtering is considered. The criterion for accuracy is the mean-square error in recovering the signal shape between readings. Linear filters with finite memory (recovery functions) of increasing complexity and the optimal linear filter, which is physically unrealizable, are considered. Calculations of the recovery errors are carried out for various reading frequencies (number of points per correlation interval) and various recovery functions. Tables and graphs are presented, which permit a comparison of the errors, evaluation of the suitability of increasing the complexity of the filter, and selection of the filter form.

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1. STATEMENT OF THE PROBLEM /3*

In communication transmission by systems with time division of channels, continuous signals undergo time discretization. The produced regular discrete sequence of readings permits recovery of the shape of the transmitted signal at the point of reception with some error, which depends on the reading frequency, the shape and width of the signal spectrum, and the recovery method. It will be assumed in the analysis below that transmitter noise and distortion in the transmission channel are small and can be neglected.

We note the magnitude of the recovery error at each moment of time:

$$E(t) = x(t) - x_r(t).$$

where $x(t)$ is the actual signal, $x_r(t)$ is the recovered signal.

Criteria for evaluating the difference in the shapes of the actual and recovered signals can differ:

1) the mean-square error in recovering the shape at any point between the discrete readings;

* Numbers in margin indicate pagination in original foreign text.

2) the probability that for some point between the readings the recovery error does not exceed a given value.

The following basic problems were solved in the work: /4

1) calculation of the recovery error in the signal shape with various correlation functions with changes in the reading frequency and the weighting function of the recovery filter;

2) comparison of the results of the calculation to produce recommendations for the selection of a method for signal recovery, i.e. an evaluation of the suitability of complicating the recovery filter to decrease error.

The calculation in the work was carried out with the first criterion.

2. SIGNAL MODELS AND SHAPE RECOVERY METHODS

We consider signals, which are steady-state random processes with given correlation (spectral) functions. It is suitable to select for these signals a single parameter ensuring the possibility of comparing the results with changes in the reading frequency. As such a parameter, we will use the process correlation interval, which we define as $T_c = 1/2F_e$, where F_e is the effective width of the signal spectrum. We define the generalized reading frequency equaling the ratio of the reading frequency to twice the effective spectral width and indicating the number of points read in the process correlation interval:

$$f = \frac{F_o}{2F_e} = \frac{T_c}{T_o} .$$

We consider random processes having the following correlation functions as power spectra as signal models:

1) white noise transmitted through a single RC filter: /5

$$R(\tau)=\sigma^2 e^{-\alpha|\tau|}, \quad S(\omega)=\frac{2\alpha}{\alpha^2+\omega^2}, \quad \alpha=4F_e;$$

2) white noise transmitted through two identical RC filters connected in series:

$$R(\tau)=\sigma^2(1+\alpha|\tau|)e^{-\alpha|\tau|}, \quad S(\omega)=\frac{4\alpha^3}{(\alpha^2+\omega^2)}, \quad \alpha=8F_e;$$

3) white noise transmitted through three RC filters in series:

$$R(\tau)=\sigma^2\left[1+\alpha|\tau|+\frac{(\alpha\tau)^2}{3}\right]e^{-\alpha|\tau|}, \quad S(\omega)=\frac{16\alpha^5}{3(\alpha^2+\omega^2)^3}, \quad \alpha=\frac{32}{3}F_e;$$

4) white noise transmitted through a filter with a gaussian frequency characteristic (the limit of a large number of RC filters connected in series):

$$R(\tau)=\sigma^2 e^{-\alpha\tau^2}, \quad S(\omega)=\sqrt{\frac{\pi}{\alpha}}\, e^{-\frac{\omega^2}{4\alpha}}, \quad \alpha=4\pi F_e^2;$$

5) white noise transmitted through a filter with a rectangular frequency characteristic with a cutoff frequency $F = F_e$:

$$R(\tau)=\sigma^2\frac{\sin 2\pi F\tau}{2\pi F\tau}.$$

We will assume that the recovery of the signal shape from the discrete readings is accomplished by a linear interpolation filter with finite memory, whose weighting function equals W(t):

$$x_r(\varepsilon)=\sum_{m=-N+1}^{N} x(mT_0)\,W(mT_0,\varepsilon),$$

where m is the number of interpolation points, M = 2N is the total /6 number of interpolation points, T_0 is the time between readings, ε is the fraction of the time interval between readings, $0\le\varepsilon\le 1$.

We consider the following methods for signal recovery between neighboring readings:

1. Stepwise recovery:

    a) without shift — the horizontal line of the estimate is drawn to the right from each sample for the time T_0;

    b) with a shift by $T_0/2$ — the horizontal line of the estimate is drawn to the right and left from each sample for the time $T_0/2$;

2. Piecewise linear recovery by connecting neighboring readings with straight lines.

3. Recovery by a finite set of functions of the form sin x/x.

4. Optimal linear recovery by a physically unrealizable filter which is used to recover an infinite number of readings on both sides of the interpolation interval.

3. MEAN-SQUARE RECOVERY ERROR CRITERION

The mean-square error in recovering the shape for each moment of time between readings can be calculated from the following formula [1]:

$$\overline{E^2(\varepsilon)} = R(0) - 2\sum_{-N+1}^{N} R[(m-\varepsilon)T_0]W[(m-\varepsilon)T_0] + \sum_{m=-N+1}^{N}\sum_{l=-N+1}^{N} R[(m-l)T_0]W[(m-\varepsilon)T_0]W[(l-\varepsilon)T_0],$$

where m, l are the number of interpolation points considered for /7
the recovery.

We will calculate the error in percent of the scale of signal change of $\pm 3\sigma(6\sigma)$:

$$E(\varepsilon) = \frac{\sqrt{\overline{E^2(\varepsilon)}}}{\sigma}.$$

The interpolation error has its maximum value at the middle of the interval between readings for all forms of interpolation, i.e. for $\varepsilon = 0.5$, except for stepwise interpolation without shift, for which it occurs for $\varepsilon = 1$).

The weighting functions of the recovery filters for these recovery methods have the following form:

1) stepwise recovery ($M = 1$, $m = 0$):

$$W(\varepsilon) = 0;$$

2) piecewise linear recovery ($M = 2$, $m = 0, 1$):

$$W[(m-\varepsilon)T_0] = \begin{cases} \varepsilon, & m = 0 \\ 1-\varepsilon, & m = 1 \end{cases}$$

3) recovery by the functions sin x/x (the number of points M is a variable quantity; recovery is produced by a filter matched with the reading frequency $F_o = 2F_e$):

$$W[(m-\varepsilon)T_0] = \frac{\sin \pi(m-\varepsilon)}{\pi(m-\varepsilon)}.$$

Determination of the error with optimal linear interpolation (averaging over all ε within the interval between readings) was carried out with the following formula obtained in [2]:

$$\overline{E^2} = \frac{1}{\pi}\int_0^\infty S(\omega)\left[1 - \frac{S(\omega)}{T_0\,\varphi(\omega)}\right]d\omega,$$ /8

where S(w) is the energy spectrum of the process to be recovered, $\varphi(\omega)=\frac{1}{T_0}\sum S(\omega-i\omega_0)$ is the energy spectrum of the discrete random process composed of the readings following with the frequency w_0.

4. MAXIMUM RECOVERY ERROR PROBABILITY CRITERION

Use of this criterion is easier for normal random processes.

Since linear filters are used for signal shape recovery, then the recovery error at each moment of time ε is a random variable with the normal probability distribution and dispersion determined by the presented formula for $\overline{E^2(\varepsilon)}$.

In connection with this, the probabiltiy that the error does not exceed a given value at each point of the interval between readings can be easily calculated with the use of normal probability distribution tables.

5. RESULTS OF THE CALCULATIONS AND CONCLUSIONS

The results of the calculation for the normalized maximum mean-square errors in signal recovery with various correlation functions are presented in the tables. /9

Table 1 contains the errors in stepwise recovery. The resulting errors are significant for any signal model even for high reading frequencies. Thus, the use of such a recovery method is unsuitable.

Table 2 and Figure 1 indicate the errors for piecewise linear recovery. The resulting errors are significantly smaller; the advantage over stepwise recovery increases with increasing reading frequency.

With the change in signal spectrum from model 1 to model 5, an ever-increasing suppression of the high-frequency "tails" occurs in the spectrum. Thus, complication of the filter shaping the signal spectrum leads to decreasing errors for any readout frequency.

Recovery by the functions $\sin x/x$ permits including a different number of points in the interpolation process without changing the weighting function of the filter. The results of calculating the errors while changing the number of points from 20 to 200 are presented in Table 3. The dependence of the errors on N is shown in Figure 2. It is interesting to note that, for the first three signal models, significantly decreasing errors are not observed for increasing numbers of points used in the recovery at the considered reading frequencies. Since there exists a limiting nonzero recovery error as $N \to \infty$, then one can draw a conclusion about the sufficiently rapid convergence of the series with the functions sin x/x. This implies that for recovery by the functions sin x/x it is not suitable in practice to use more than 10 - 20 interpolation points. If one considers the change in the errors with increasing reading frequency and constant number of points from Figure 3, then it will be noted that increasing the reading frequency above some value leads to a constant error, which has the /10
same value independent of the signal model.

Comparison of the piecewise linear recovery and recovery by the functions sin x/x can be carried out on Figure 4 and 5. The errors for the optimal linear, piecewise linear recovery and recovery by the functions sin x/x averaged over all values of ε in the interval between readings are presented in Figure 6. The comparison indicates that it is not suitable to complicate the recovery filters up to the spectral shape determined by the forming 3 x RC filter, since the use of the piecewise linear recovery provides practically the same magnitude of the errors as the optimal linear recovery. Only for the gaussian shape of the signal spectrum are the errors of the optimal linear recovery smaller in comparison with the piecewise linear recovery. The difference increases with high reading frequencies and small errors. With increasing accuracy requirements, such a difference in errors can be significant. In this case recovery by the functions sin x/x permits a significant decrease of this difference up to some reading frequency depending on the number of interpolation points used.

It is interesting to note that recovery by the functions sin x/x leads to errors practically coinciding with the errors of the optimal linear recovery for an unknown signal model, but belonging to the considered class.

This also permits one to draw the conclusion about the practical unsuitability of analyzing and using the optimal linear recovery over a finite number of interpolation points.

In practice, the small difference in errors produced by the piecewise linear recovery in comparison to the more complicated recovery methods leads to the conclusion about the suitability of the preferred use of the piecewise linear recovery of signal between readings. /11

REFERENCES

1. Andreyev, N.I. Korrelyatsionnaya teoriya statisticheski optimal'nykh sistem (Correlation theory of statistically optimal systems). Moscow, "Nauka" Press, 1966.

2. Bykov, V.V. Tsifrovoye modelirovaniye v statisticheskoy radiotekhnike (Digital simulation in statistical radio engineering). Moscow, "Sovyetskoye radio" Press, 1971.

TABLE 1. STEPWISE RECOVERY*

Recovery Form \ f	1	5	10	20	50	100	Form of correlation function
without shift	22	13,5	10	7,25	4,7	3,33	RC filter
with shift	19,8	10	7,25	5,16	3,33	2,33	RC filter
without shift	22,5	10,3	5,8	3,1	1,3	0,7	2×RC filter
with shift	18,2	5,8	3,1	1,6	0,7	0,33	2×RC filter
without shift	22,7	9,4	5	2,6	1	0,5	3×RC filter
with shift	18	5	2,6	1,3	0,5	0,26	3×RC filter
without shift	23	8	4,1	2,1	0,84	0,42	Gauss. filter
with shift	17,4	4,1	2,1	1	0,42	0,21	Gauss. filter
without shift	23,6	6	3	1,6	0,6	0,3	Rect. filter
with shift	14,2	3	1,6	0,8	0,3	0,16	Rect. filter

TABLE 2. LINEAR RECOVERY*

Form of correlation function \ f	1	2	3	5	10	20	50	100
RC-filter	15,2	11,8	9,8	7,4	5,5	3,7	2,3	1,67
2×RC-filter	14,3	8,02	5,17	2,77	1,09	0,4	0,1	0,04
3×RC-filter	13,94	6,82	3,88	1,72	0,5	0,14	0,023	0,006
Gauss. filter	13,01	4,85	2,35	0,88	0,23	0,057	0,009	0,0023
Rect. filter	7,94	2,22	1,0	0,37	0,09	0,023	0,0036	0,0008

* Translator's note: Commas in numbers represent decimal points.

TABLE 3. RECOVERY BY THE FUNCTIONS sin x/x*

N \ f	1	2	3	5	10	20	Form of correlation function
10	17,4	12,9	10,6	8,3	5,9	4,2	RC filter
20	17,43	12,9	10,51	8,31	5,91	4,21	
50	17,45	12,9	10,64	8,33	5,92	4,22	
100	17,5	12,96	10,65	8,35	5,93	4,23	
10	16,2	8,7	5,27	2,48	1,08	0,5	2×RC filter
20	16,3	8,78	5,32	2,42	0,98	0,39	
50	16,35	8,8	5,33	2,42	0,96	0,35	
100	16,36	8,8	5,33	2,42	0,96	0,34	
5	15,87	6,95	3,4	1,36	0,87	–	3×RC filter
10	16,02	7,03	3,4	1,19	0,42	–	
20	16,1	7,08	3,42	1,16	0,28	–	
10	15,1	3,06	0,59	0,39	0,38	0,37	Gauss. filter
20	15,15	3,5	0,47	0,19	0,189	0,186	
50	15,2	3,6	0,45	0,08	0,07	0,04	
100	15,25	3,67	0,43	0,039	0,038	0,037	
10	2,37	0,42	0,4	0,38	0,37	0,37	Rect. filter
20	1,67	0,21	0,2	0,19	0,187	0,186	
50	1,05	0,083	0,08	0,08	0,08	0,08	
100	0,78	0,042	0,041	0,038	0,037	0,037	

* Translator's note: Commas in numbers represent decimal points.

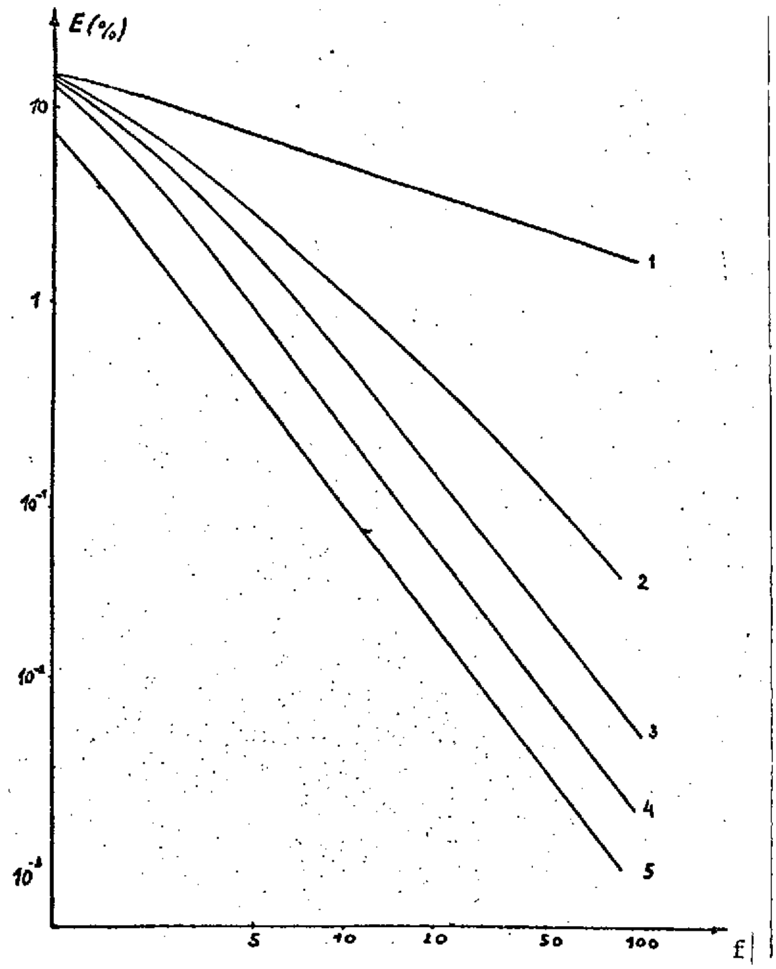

Figure 1. Errors for piecewise linear recovery (P.L.)

1- RC filter; 2- 2 x RC filter; 3- 3 x RC filter; 4- gaussian filter; 5- rectangular filter

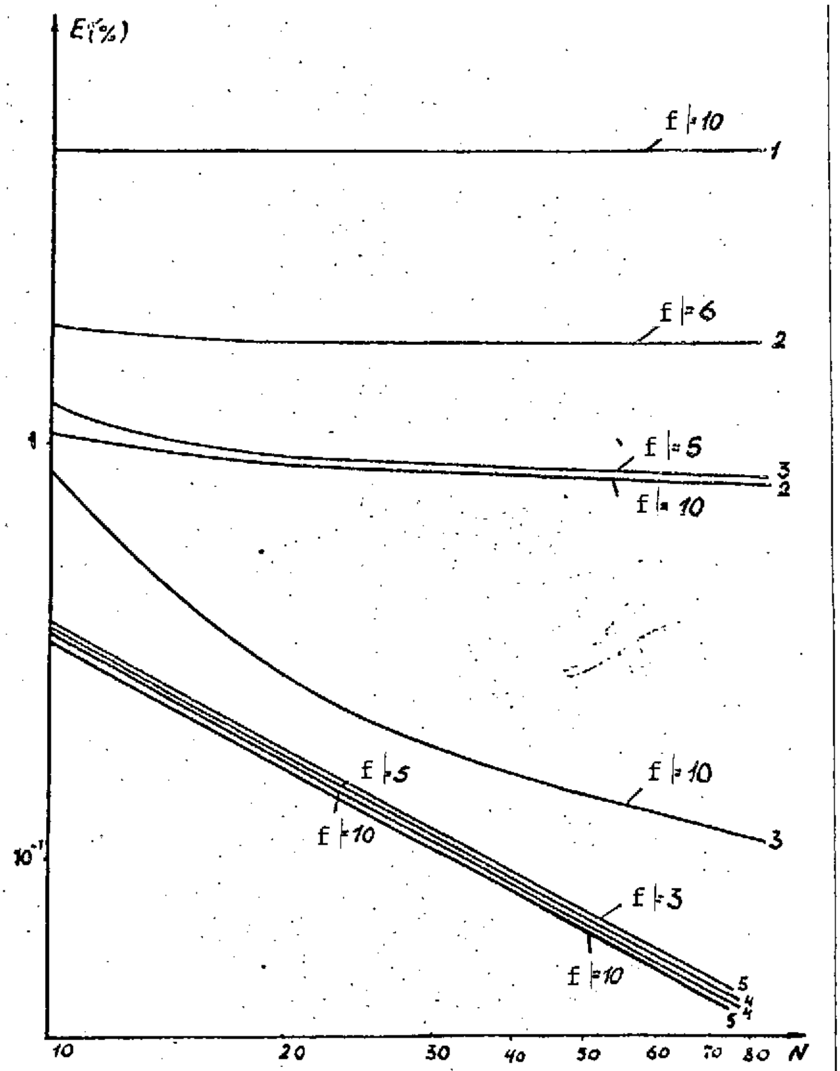

Figure 2. Errors for recovery by the functions sin x/x with changing number of points (M = 2N)

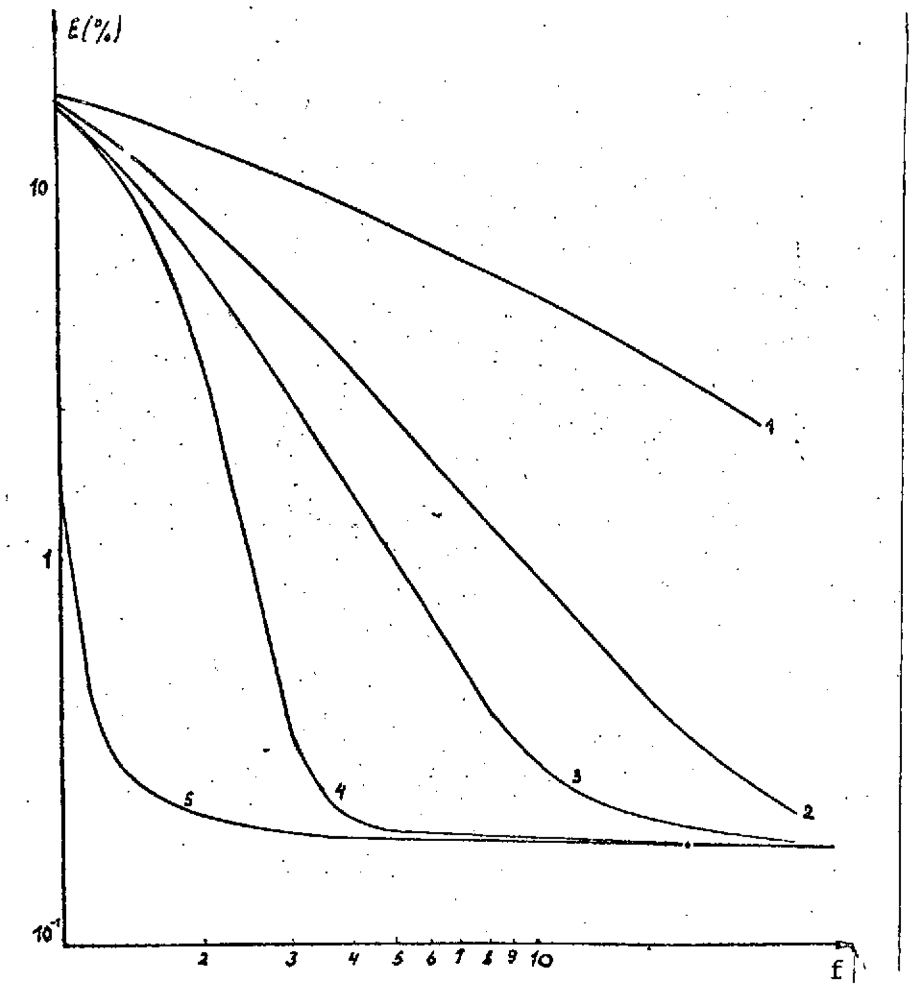

Figure 3. Errors for recovery by the functions sin x/x, N = 20
1- RC filter; 2- 2 x RC filter; 3- 3 x RC filter; 4- gaussian filter; 5- rectangular filter

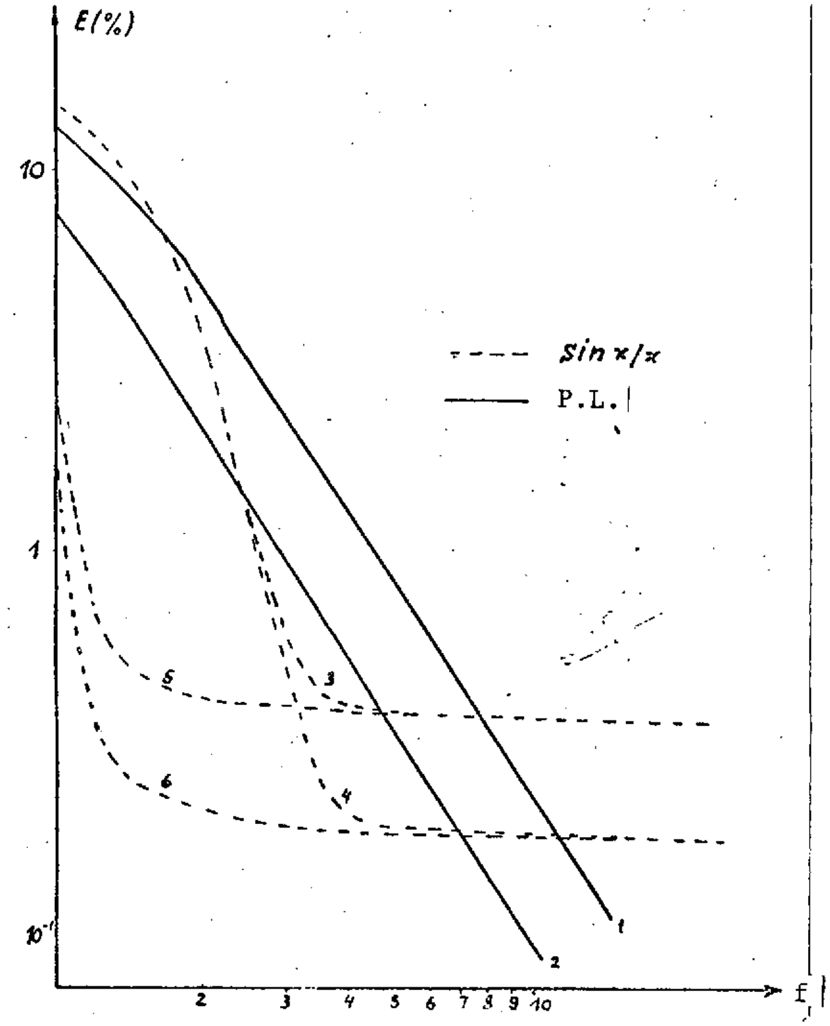

Figure 4. Comparison of recovery methods

1, 3, 4- gaussian filter; 2, 5, 6- rectangular filter; 3, 5- N = 10; 4, 6- N = 20

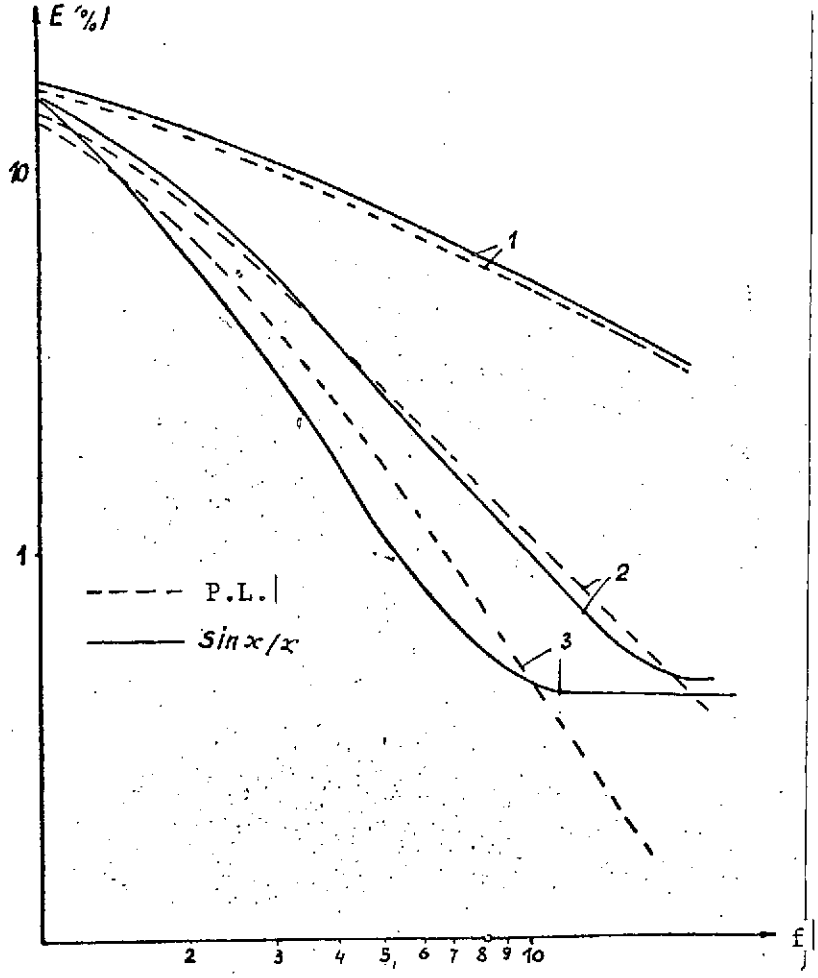

Figure 5. Comparison of recovery methods
1- RC filter; 2- 2 x RC filter; 3- 3 x RC filter; N = 10

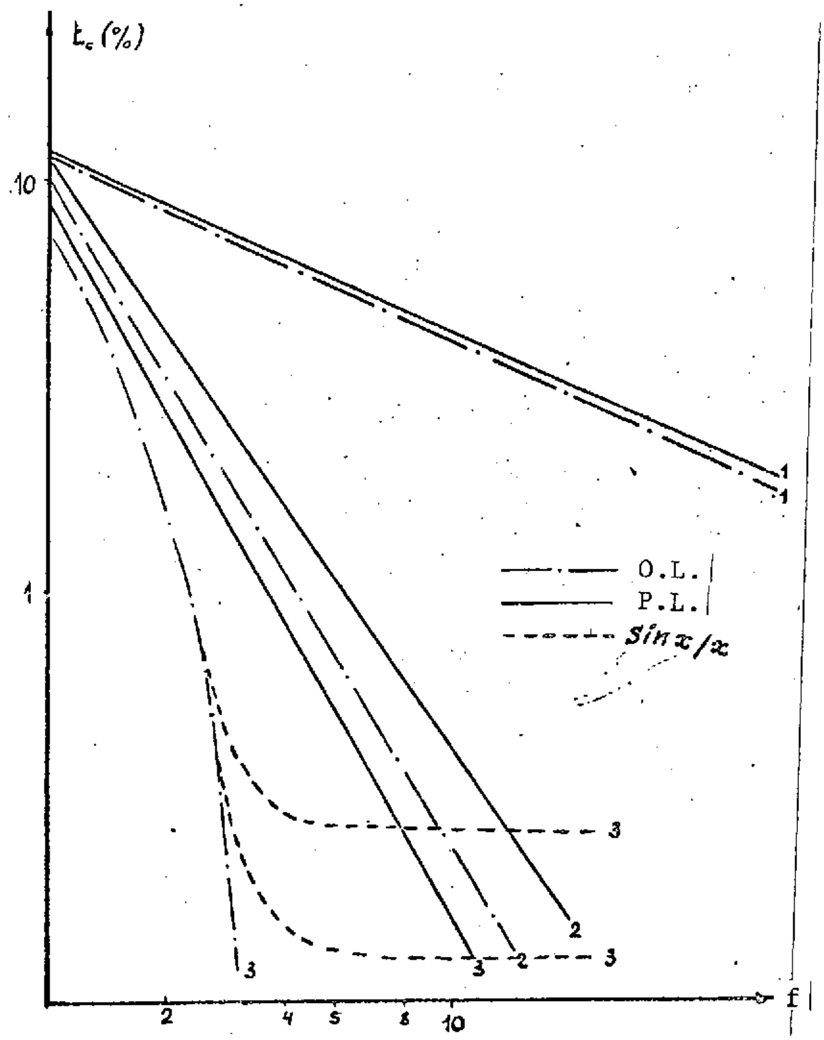

Figure 6. Comparison with optimal linear recovery (O.L.)
1- RC filter; 2- 3 x RC filter; 3- gaussian filter

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16. Abstract

Linear filters with finite memory of increasing complexity and the optimal linear filter, which is physically unrealizable, are considered. Calculations of the recovery errors are carried out for various reading frequencies and various recovery functions.

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